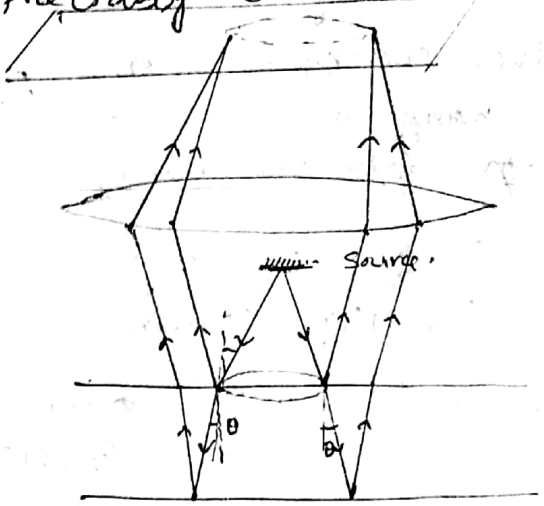
Haidinger fringes

$\Delta = 2\mu e \cos\theta$ Δ depends on θ and e.

$\delta(\Delta) = 2\mu e\, \delta(\cos\theta) + 2\mu \cos\theta\, \delta(e)$

If e remains const (for parallel film), $\delta(\Delta)$ depends on $\delta(\cos\theta)$, when e is large, Δ changes appreciably even when θ changes by a small amount.



- Fringes are produced due to superposition of rays equally inclined to the normal,

Fringes of equal inclination.

Called "Haidinger Fringes".

Fringes are formed at infinity.

- Needs an extended source.
- Fringes are circular in shape.

Fizeau fringes.

- Fringes of equal thickness formed by inclined film.
- Localized fringes.

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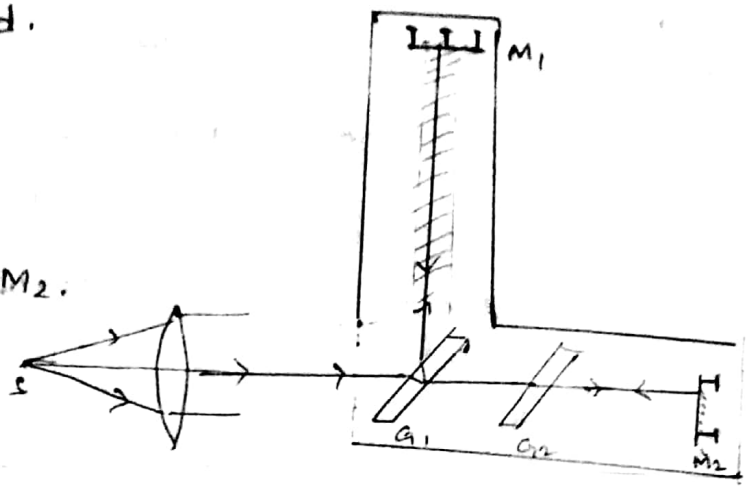
Michelson's Interferometer.

Interferometer : an instrument.

Construction :



Phase change on reflection :

Phase change by π at M_1 & M_2.

If silver coating is thin, phase change π occurs for both the waves.

$\Delta = 2n\frac{\lambda}{2}$ (Maxima)

$\Delta = (2n+1)\lambda/2$ (Minima). } Partially silvered.

Unsilvered $\Delta = 2n\lambda$ (Minima)

$\Delta = (2n+1)\frac{\lambda}{2}$ (Maxima).

M_2' : image of M_2 on G_1.

Equivalent to a thin air film.

If $\phi = 0$ but $e \neq 0$: Circular fringes.

If $e = 0$ but $\phi \neq 0$, Sl. fringes are convex towards intersection.

Form of fringes

S' : image of real source S by G_1.

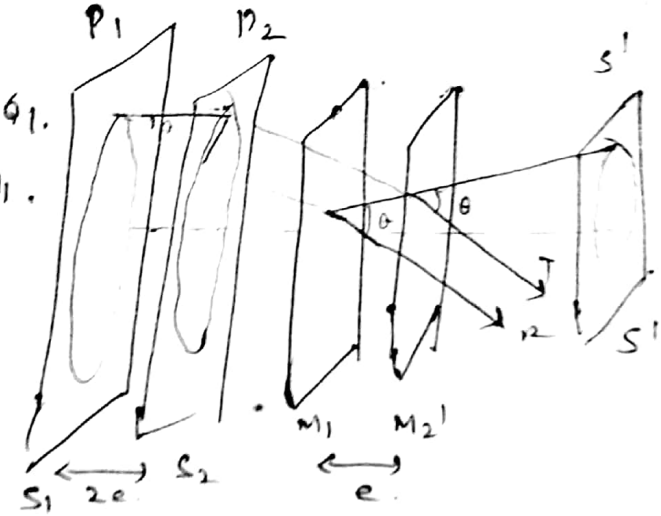
S_1 & S_2 are images of S' by M_2' & M_1.

$M_1 M_2' = e$, $S_1 S_2 = 2e$.

P_1 & P_2 are images of P.

$\Delta = 2R\cos\theta = n\lambda$: Maxima.

For same e, n, λ, θ remains const.

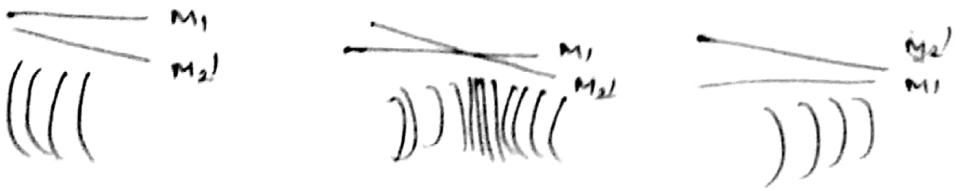


Fringes are circle,

Fringes are situated at ∞.

Fringes number increases towards centre.

Localised fringes.



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Application

a) determination of λ :

$$2e = m\lambda \quad , \quad 2(e + \lambda/2) = (m+1)\lambda$$

$$\frac{1}{2} N\lambda = x_2 - x_1$$

b) $\lambda_1 - \lambda_2$ & $\frac{\lambda_1}{\lambda_2}$

$$\left.\begin{aligned} 2d_0 &= n_1\lambda_1 \\ 2d_0 &= n_2\lambda_2 \end{aligned}\right\} \quad \begin{aligned} 2(d_0 + d) &= (n_1 + N)\lambda_1 \\ 2(d_0 + d) &= (n_2 + N + 1)\lambda_2 \end{aligned}$$

$$2d = N\lambda_1 \qquad 2d = (N+1)\lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{N+1}{N} \Rightarrow \frac{\lambda_1 - \lambda_2}{\lambda_2} = \frac{1}{N} = \frac{\lambda_1}{2d}$$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2d} = \frac{\lambda_{av}^2}{2d}$$

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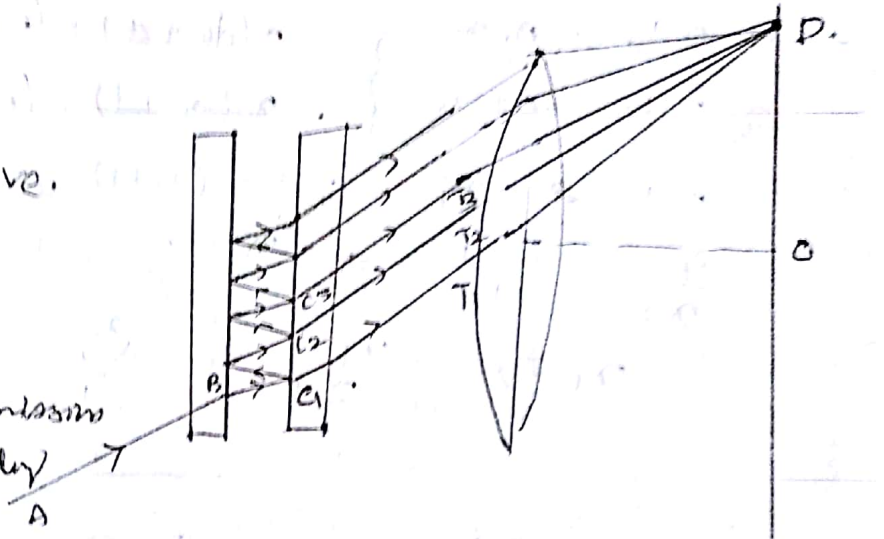
Fabry - Perot interferometer

- sharp resolving power.
- can see the fine structure of spectral line.
- Two glasses or quartz plates, inner surface is partly silvered.
- Distance between them can be moved by micrometer.
- Mechanical problem : etalon : separated by spacer : (1 to 20 mm)

Theory.



Intensity distribution.

A_i = amplitude of incident wave.

C_1T_1, C_2T_2, C_3T_3 .... are transmitted waves.

r, t : reflection and transmission co-efficient from surrounding to the medium.

r', t' = reflection and transmission co-efficient from the medium to the surrounding.

Amplitude of successive transmitted ray can be written as

$$tt' A_i \;,\; tt' r'^2 A_i e^{i\delta} \;,\; tt' r'^4 A_i e^{2i\delta} \;,\; tt' r'^6 A_i e^{3i\delta} \;\; tt' r'^{2(n-1)} A_i e^{i(n-1)\delta} \;\; \ldots$$

$$Y = A_i e^{-i(\omega t - kx)}$$

$$Y = A_i e^{-i(\omega t - kx + k\Delta)} = A_i e^{-ik\Delta} e^{-i(\omega t - kx)} \qquad k\Delta = \delta$$

We have $tt' = 1 - r^2 = T$ = transmissivity.

$$r = -r', \quad r^2 = r'^2 = R = \text{Reflectivity}$$

$$R + T = 1.$$

Total amplitude

$$A_t = tt' \left[1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \ldots \right] A_i$$

$$= \frac{tt' A_i}{1 - R e^{i\delta}}$$

$$I_t = A_t A_t^* = \frac{T}{1 - R e^{i\delta}} \cdot \frac{T}{1 - R e^{-i\delta}} A_i^2 = \frac{T^2}{(1 - R e^{i\delta})(1 - R e^{-i\delta})} A_i$$

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$$\frac{I_t}{I_i} = \frac{T^2}{1+R^2-2R\cos\delta} = \frac{T^2}{(1-R)^2+4R\sin^2\delta/2}$$

$$= \frac{T^2}{(1-R)^2}\left[\frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\delta/2}\right]$$

If $\delta = 2n\pi$ or $\Delta = n\lambda$, I_t is maximum: $I_{max} = \frac{T^2}{(1-R)^2} I_i$

If $\delta = (2n+1)\pi$ or $\Delta = (2n+1)\frac{\lambda}{2}$, I_t is minimum, $I_{min} = \frac{T^2}{(1+R)^2} I_i$

If there is no absorption of light by the film, then

$$T = 1-R$$

$I_{max} = I_i$ and $I_{min} = \frac{(1-R)^2}{(1+R)^2} I_i$, $I_{min} \neq 0$.

Visibility of fringes

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2R}{1+R^2}$$

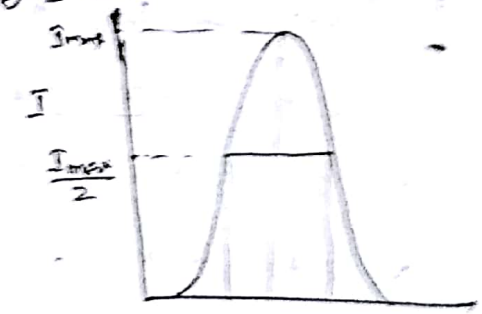
It is independent of transparency of the film.

Sharpness of fringes

$$I = \frac{I_{max}}{1+F\sin^2\delta/2}$$ Where $F = \frac{4R}{(1-R)^2}$

= Co-efficient of fineness.

F determines the sharpness of the fringes.



Fringe half-width

Width of the fringes where the intensity becomes half of I_{max}.

When $\delta = \delta'$, $I = \frac{I_{max}}{2}$

$\therefore 1 + F\sin^2\delta/2 = 2 \Rightarrow F\sin^2\delta/2 = 1$

$\therefore \sin\frac{\delta'}{2} = \pm\frac{1}{\sqrt{F}}$

when R is large (more than ½), F is large, $\frac{1}{\sqrt{F}}$ is small

hence $\sin\frac{\delta'}{2} = \frac{\delta'}{2}$

$\therefore \delta' = \pm\frac{2}{\sqrt{F}}$

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For 2π phase diff, width of the fringe = 1.

For $2\delta'$ phase diff, width of the fringe = half width.

$\therefore$ Half width = $W = \frac{2\delta'}{2\pi} = \frac{\delta'}{\pi} = \frac{2}{\pi\sqrt{F}} = \frac{1-R}{\pi\sqrt{R}}$

W depends on F.

The more is the fineness (F), the less is W, the more is the sharpness.

For Michelson interferometer, $\delta' = 1.57$ radian

For Fabry-Perot interferometer when $R = 0.9$, $\delta' = 0.105$

So the fringes of Fabry-Perot interferometer is sharp.

Shape of fringes

Condition of getting bright fringes is

$$2e\cos\theta = n\lambda \quad \text{as } \mu = 1.$$

For a given value of e, λ and n, θ = constant.

So the fringes are circular.

Determination of wavelength

Exactly like Michelson interferometer.

$$\frac{1}{2}N\lambda = x_2 - x_1$$

Difference of two close wavelengths.

$$2d_0 = n_1\lambda_1 = n_2\lambda_2$$

& $2(d_0 + d) = (n_1 + N)\lambda_1 = (n_2 + N + 1)\lambda_2$

$\therefore 2d = N\lambda_1 = (N+1)\lambda_2$

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{N+1}{N} \Rightarrow \frac{\lambda_1 - \lambda_2}{\lambda_2} = \frac{1}{N} = \frac{\lambda_1}{2d}$

$$\lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2d} = \frac{\lambda_{av}^2}{2d}.$$

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